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Invisible decays of Higgs and other mesons in models with singlet neutrinos in large extra dimensions

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Abstract

In light of current atmospheric neutrino oscillation data, we revisit the invisible decay of the standard model Higgs boson and other pseudoscalar mesons which can be enhanced because of large number of KK modes in models with right-handed singlet neutrinos in large extra dimensions. We find that the invisible decay rate of Higgs can be as large as $H \rightarrow b\bar{b}$ decay rate only for a very restricted region of parameter space. This parameter space is even further restricted if one demands that the dimensionless neutrino Yukawa coupling λ is $O(1)$. We have also studied the scenarios where singlet neutrino propagate in a sub-space, which lowers the string scale M_* and keeps neutrino Yukawa coupling $O(1)$. We have also considered decays of other spin-0 mesons to $\nu\bar{\nu}$ and found the rates to be too small for measurement.

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1. Introduction

The concept of large extra dimensions and TeV scale quantum gravity [1] introduced by ADD has attracted a lot of attentions recently. In this scenario, one has δ additional spatial dimensions of size R in which gravity propagates whereas the standard model particles with chiral fermionic content are confined to the usual 4 dimensions (4D). The effective 4D Planck scale, $M_{\text{Pl}} \sim 2.4 \times 10^{18}$ GeV, is related to the *fundamental* Planck scale M_* in $4 + \delta$ dimension by

$$M_{\text{Pl}}^2 \sim R^\delta M_*^{\delta+2}. \quad (1)$$

Thus, for the large extra dimensions, it is possible to have a *fundamental* scale M_* as low as a TeV [1], helping to resolve the gauge hierarchy problem of the standard model. The size of the extra dimensions can be as large as $\sim \text{mm}$ for $\delta = 2$. Another interesting aspect of this scenario is the generation of a small neutrino mass [2,3]. The relatively small value of M_* indicates the possibility of a seesaw mechanism, with right-handed (RH) singlet neutrinos also propagating in the full $4 + \delta$ dimensions with gravity [4–9]. For illustration let us assume the case of a single extra-dimension labeled by y (x labels the usual 4D) [10].

A massless Dirac fermion N which is a standard model singlet lives in 5D. The Γ matrices in 5D can be written as

$$\Gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \Gamma^5 = \begin{pmatrix} i & 0 \\ 0 & -i \end{pmatrix}. \quad (2)$$

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The Dirac spinor N in the Weyl basis can be written as

$$N = \begin{pmatrix} \psi \\ \bar{\chi} \end{pmatrix}, \quad (3)$$

where ψ and χ are 2-component complex spinors (with mass dimension 2). The 5D kinetic term for N is

$$S_{\text{free}} = \int d^4x dy \bar{N} (\Gamma^\mu \partial_\mu + \Gamma^5 \partial_y) N \quad (4)$$

and the interaction action is

$$S_{\text{int}} = \int d^4x \frac{\lambda}{\sqrt{M_*}} l(x) H^*(x) \psi(x, y=0), \quad (5)$$

where, $\ell = (e, \nu)$ is the standard model lepton doublet and H is the Higgs doublet and λ is a dimensionless Yukawa coupling in 5D. We have assigned N and ℓ opposite lepton number, so that the lepton number is conserved in Eq. (5).

In the effective 4D theory, N appears as a tower of Kaluza–Klein (KK) states:

$$\psi = \sum_n \frac{1}{\sqrt{R}} \psi^{(n)}(x) e^{iny/R}, \quad (6)$$

where $\psi^{(n)}$ are 4D states. Similarly, χ has a KK tower, $\chi^{(n)}$. Thus, in 4D, we have the effective action:

$$S_{\text{free}} = \int d^4x \sum_n \left[\bar{\psi}^{(n)} \bar{\sigma}^\mu \psi^{(n)} + \bar{\chi}^{(n)} \bar{\sigma}^\mu \chi^{(n)} + \left(\frac{n}{R} \psi^{(n)} \chi^{(n)} + \text{h.c.} \right) \right], \quad (7)$$

$$S_{\text{int}} = \int d^4x \sum_n \lambda \frac{M_*}{M_{\text{Pl}}} l(x) H^*(x) \psi^{(n)}(x), \quad (8)$$

where n/R is the Dirac mass for the KK states.

Thus, we see that the 4D neutrino Yukawa coupling is suppressed by the volume of the extra dimensions, or in other words, by the ratio M_*/M_{Pl} . When H gets a VEV ($v \approx 246$ GeV), we get Dirac mass for standard model neutrino (ν) denoted by m :

$$m \approx \frac{\lambda}{\sqrt{2}} \frac{M_*}{M_{\text{Pl}}} v \sim \lambda \frac{M_*}{\text{TeV}} 10^{-4} \text{ eV}. \quad (9)$$

However, this standard model neutrino (which is dominantly the lightest neutrino) has a small mixture ($\sim mR/n$) of heavier neutrinos. With this, the lightest neutrino mass is modified to (we extend the formula to

δ extra dimensions)

$$m_\nu \approx \frac{m}{\sqrt{1 + \frac{m^2}{M_*^2} \frac{M_{\text{Pl}}^2}{M_*^2} \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)(\delta-2)}}}. \quad (10)$$

Now, for a given value of M_* and δ the upper limit of m_ν is $m_\nu^{\text{max}} \approx \frac{M_*^2}{M_{\text{Pl}}^2} \sqrt{\frac{\Gamma(\delta/2)(\delta-2)}{2\pi^{\delta/2}}}$, (for $\delta = 2$, $\delta - 2$ is replaced by $1/\ln(M_{\text{Pl}}/M_*)$). We then get

$$m_\nu \approx \frac{m}{\sqrt{1 + \left(\frac{m}{m_\nu^{\text{max}}}\right)^2}}. \quad (11)$$

2. Invisible decay of Higgs

It has been shown in Ref. [11] that the standard model Higgs can decay into $\nu_L \bar{\nu}_R^i$ with the strength $\sim \frac{\lambda}{\sqrt{2}} \frac{M_*}{M_{\text{Pl}}}$, which looks like rather a small number. However, one can get large enhancement in the decay rate when one sums the KK excitations of right-handed neutrino states with masses below the Higgs mass (M_H). This summation is proportional to the volume R^δ of the extra-dimensional space, with a momentum-space factor of order $(M_H R)^\delta$. So, the rate is enhanced by the factor $(M_H R)^\delta$. Therefore, the sum of partial widths of the Higgs into KK excitations of neutrinos is of the order of:

$$\sum_i \Gamma(H \rightarrow \nu_L \bar{\nu}_R^i) \sim \frac{M_H}{16\pi} Y_\nu^2 (M_H R)^\delta \quad (12)$$

where, $Y_\nu \sim \frac{\lambda}{\sqrt{2}} \frac{M_*}{M_{\text{Pl}}}$. To quantify the invisible decay rate of the Higgs boson, following the notation of Ref. [11], we define the ratio $\kappa (\equiv \sum_i B(H \rightarrow \nu_L \bar{\nu}_R^i) / B(H \rightarrow b\bar{b}))$, which can be expressed as:

$$\kappa \simeq \frac{m^2}{3m_b^2} \left(\frac{M_H}{M_*} \right)^\delta \left(\frac{M_{\text{Pl}}}{M_*} \right)^2. \quad (13)$$

From this expression it is clear that κ increases with m and M_H , while it decreases with increase in δ and M_* . Note that κ depends on m rather than m_ν directly.

We now evaluate the ratio κ in light of the latest atmospheric neutrino oscillation data, which suggest $\Delta m_{\text{atm}}^2 = (1.5\text{--}4.0) \times 10^{-3} \text{ eV}^2$ [12]. In our analysis we fix the highest neutrino mass which dominates $H \rightarrow \nu \bar{\nu}$ rate to be $m_\nu \sim \sqrt{\Delta m_{\text{atm}}^2}$ and then solve

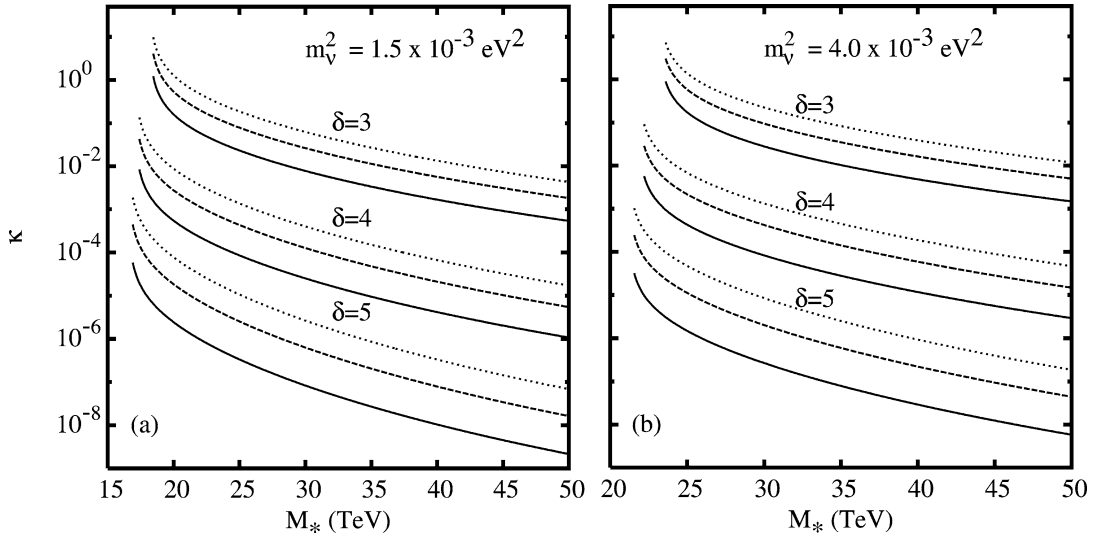


Fig. 1. Variation of κ (defined in the text) with M_* for three values of extra dimensions $\delta = 3, 4$ and 5 . The solid, broken and dotted lines correspond to $M_H = 100, 150$ and 200 GeV, respectively. (a) and (b) correspond to $m_\nu^2 = 1.5 \times 10^{-3} \text{ eV}^2$ and $4.0 \times 10^{-3} \text{ eV}^2$, respectively.

for m from Eq. (10) in terms of M_* and δ . The see-saw mechanism in Eq. (9) clearly favors hierarchical neutrino masses rather than degenerate mass spectrum (which will need fine tuning of λ). We consider two cases that respect the allowed range of Δm_{atm}^2 , (a) $m_\nu^2 = 1.5 \times 10^{-3} \text{ eV}^2$ and (b) $m_\nu^2 = 4.0 \times 10^{-3} \text{ eV}^2$. Another important consideration is the value of λ that ensues for a given m and M_* . Clearly, the value of λ , which is the higher-dimensional Yukawa coupling, should not exceed ~ 10 – 20 , otherwise we will be in a non-perturbative regime. This is a serious constraint on allowed value of M_* for a given δ .

In Fig. 1(a) we show the variation of κ with M_* for three choices of extra dimensions $\delta = 3, 4$ and 5 keeping $m_\nu^2 = 1.5 \times 10^{-3}$. The solid, broken and dotted curves correspond to $100, 150$ and 200 GeV Higgs mass, respectively. Left end of each curve has been truncated by using a constraint $m \leq 4m_\nu$. Value of m higher than this correspond to unacceptably large λ . From this figure, it is clear that for $\delta = 4$ and 5 the invisible width is only a tiny fraction of $H \rightarrow b\bar{b}$ rate. For $\delta = 3$, the value of κ is at most $1, 3$ or 7 for the Higgs masses $100, 150$ and 200 GeV respectively, and less than 1% for most of the parameter space.

Similar variation is shown in Fig. 1(b), however, for $m_\nu^2 \sim 4.0 \times 10^{-3}$. The situation is better in this case,

since, higher value of m_ν lead to larger value of m in the ratio κ . In this case also we have truncated each curve on the left by imposing the same condition as in Fig. 1(a).

Before we proceed, we would like to mention two points here. Firstly, beyond $M_H = 150$ GeV, the dominant mode of Higgs decay is $H \rightarrow WW^*$, thus the $H \rightarrow b\bar{b}$ branching ratio is very small, and any $H \rightarrow \nu\bar{\nu}$ branching ratio will also become small. However, for the purpose of comparison we have presented the case of 200 GeV Higgs mass. One should also note that for the value of M_* at which κ is $O(1)$ and larger, corresponding Yukawa coupling λ is around 80 for $\delta = 3$. Such a large value of λ may not be accepted from the perturbative point of view.

For a given value of m_ν and M_* , the value of λ can be obtained from Eqs. (9) and (10):

$$\lambda = \frac{(m_\nu/M_*)10^{16}}{\sqrt{1 - \frac{m_\nu^2}{M_*^2} \frac{M_{\text{Pl}}^2}{M_*^2} \frac{2\pi^{\delta/2}}{\Gamma(\delta/2)(\delta-2)}}}. \quad (14)$$

It is easy to show that if we limit $\lambda \leq 10$ (to be in perturbative regime), for $m_\nu^2 = 1.5 \times 10^{-3} \text{ eV}^2$ we are restricted to $M_* \geq 40 \text{ TeV}$ and for $m_\nu^2 = 4.0 \times 10^{-3} \text{ eV}^2$, $M_* \geq 64 \text{ TeV}$. The dependence on δ is very weak on these bounds. Thus for such high values of M_* the ratio κ is highly suppressed.

For completeness we discuss the case of degenerate neutrinos, separated by Δm^2 consistent with the atmospheric and solar neutrino data. Recently WMAP [13] provided important new information on cosmic microwave background anisotropies. After combining the data from 2dF Galaxy Redshift Survey, CBI, and ACBAR [14], WMAP places stringent limits on the contributions of neutrinos to the energy density of the universe [15]

$$\Omega_\nu h^2 = \frac{\sum_i m_{\nu_i}}{93.5 \text{ eV}} < 0.0076 \quad (95\% \text{ C.L.}), \quad (15)$$

which implies

$$\sum_i m_{\nu_i} < 0.71 \text{ eV} \quad (16)$$

for a single active neutrino, or $m_\nu < 0.23 \text{ eV}$ for three degenerate neutrinos. Using the value $m_\nu = 0.23 \text{ eV}$ and multiplying Eq. (13) by 3 for three families we can calculate κ . This is shown in Fig. 2. The general behavior of κ as a function of M_* is similar to Fig. 1. The only point to be noted here is that, because of heavier neutrino mass, the allowed range of M_* is much higher than the previous case. If we now impose the condition that $\lambda \leq 10$ to satisfy the perturbative condition, M_* becomes too heavy $\sim 200 \text{ TeV}$. For such a large value of M_* , κ drops below 10^{-4} for $M_H = 100 \text{ GeV}$ and $\delta = 3$, and is essentially unmeasurable. The situation is worse for higher values of δ .

In summary, we see from Figs. 1(a), (b) and 2 that κ of the order $O(1)$ can arise only when $M_* \sim 20\text{--}30 \text{ TeV}$. However, as noted, this implies $\lambda > 70$, making the theory non-perturbative. There are two ways by which one can evade this problem. Either one should take $\lambda \sim O(1)$, in that case, M_* will be $\geq 100 \text{ TeV}$ for $m_\nu \sim \sqrt{\Delta m_{\text{atm}}^2}$ and $M_* \geq 200 \text{ TeV}$ for $m_\nu = 0.23 \text{ eV}$. This is perfectly fine, but for such a large value of M_* , the invisible decay width of Higgs will be negligibly small compared to $H \rightarrow b\bar{b}$. The second choice suggested by [3,16] is to consider that the singlet neutrino propagates in a sub-space (δ_ν) of the full extra dimension (δ) where gravity propagates. Assuming that all extra dimensions are of the same size R , in this case, the Dirac mass for the standard model neutrino now becomes

$$m \sim \lambda v \left(\frac{M_*}{M_{\text{Pl}}} \right)^{\delta_\nu/\delta}. \quad (17)$$

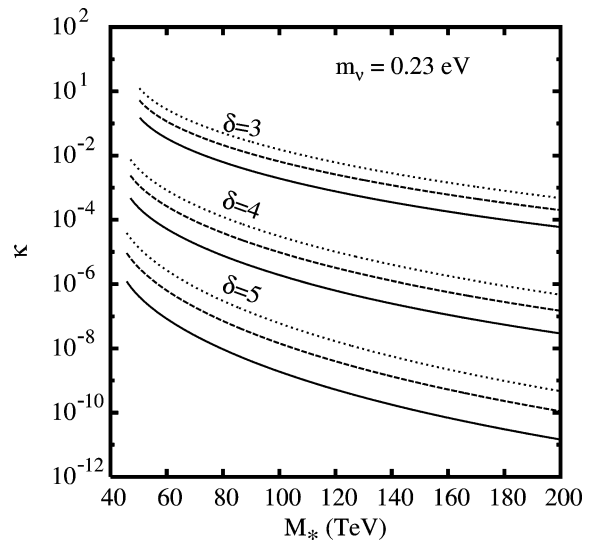


Fig. 2. Variation of κ with M_* for three values of extra dimensions $\delta = 3, 4$ and 5 . The solid, broken and dotted lines correspond to $M_H = 100, 150$ and 200 GeV , respectively. We have fixed $m_\nu = 0.23 \text{ eV}$.

Thus for $\delta_\nu = 5$ and $\delta = 6$ and with $M_* \sim \text{TeV}$, $\lambda \sim O(1)$, we obtain $m^2 \sim \Delta m_{\text{atm}}^2$. It has been shown by Agashe and Wu [16] that for the above choices of δ_ν and δ , the constraints on M_* from $\text{BR}(\mu \rightarrow e\gamma)$ and $\pi \rightarrow e\bar{\nu}, \mu\bar{\nu}$ decays can be significantly weakened as compared to the minimal model, allowing the scale $M_* \sim \text{TeV}$. In this case, the maximum value of the physical neutrino mass for a given δ_ν, δ and M_* is given by [16]:

$$m_\nu^{\text{max}} \approx M_* \left(\frac{M_*}{M_{\text{Pl}}} \right)^{\delta_\nu/\delta} \sqrt{\frac{\Gamma(\delta_\nu/2)(\delta_\nu - 2)}{2\pi^{\delta_\nu/2}}}. \quad (18)$$

Following Eq. (18), one can invert Eq. (11) to get the neutrino mass parameter m which enters in the Higgs decay width. In sub-space, by replacing $\delta \rightarrow \delta_\nu$ and $(M_{\text{Pl}}/M_*)^2 \rightarrow (M_{\text{Pl}}/M_*)^{2(\delta_\nu/\delta)}$ one can get the ratio κ in terms of the neutrino mass parameter m , δ_ν , δ , and M_*

$$\kappa \simeq \frac{m^2}{3m_b^2} \left(\frac{M_H}{M_*} \right)^{\delta_\nu} \left(\frac{M_{\text{Pl}}}{M_*} \right)^{2(\delta_\nu/\delta)}. \quad (19)$$

We now compute the ratio κ as a function of M_* keeping $\delta_\nu = 5$ and $\delta = 6$. In Fig. 3(a) and (b), we show κ as a function of M_* for $m_\nu^2 = 1.5 \times 10^{-3} \text{ eV}^2$ and $4.0 \times 10^{-3} \text{ eV}^2$, respectively. The choice of the

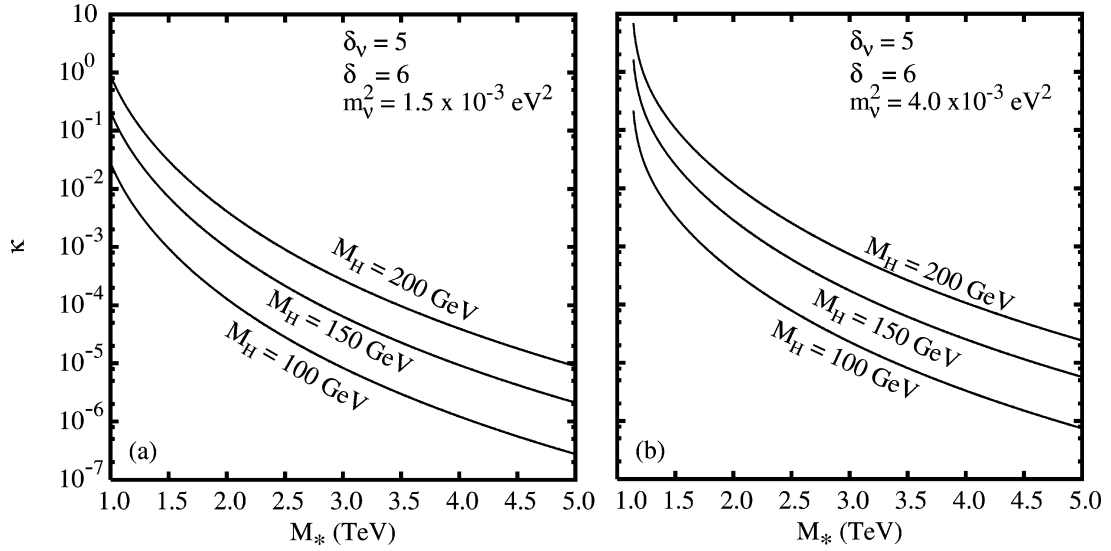


Fig. 3. Variation of κ with M_* for number of sub-space extra dimensions $\delta_\nu = 5$, number of extra dimensions $\delta = 6$ and $M_H = 100, 150$ and 200 GeV. (a) and (b) correspond to $m_\nu^2 = 1.5 \times 10^{-3} \text{ eV}^2$ and $4.0 \times 10^{-3} \text{ eV}^2$, respectively.

Higgs masses is the same as before. Comparing Fig. 3 with the earlier figures, it is clear that M_* can now be as low as 1 TeV. This value also is allowed by other experimental constraints [16]. From Fig. 3(a), one can see that $\kappa \sim 1$ only for $M_H = 200$ GeV and at $M_* = 1$ TeV. As M_* increases κ decreases significantly. While in Fig. 3(b), which corresponds to the higher value of the physical neutrino mass, $m_\nu^2 = 4.0 \times 10^{-3} \text{ eV}^2$, κ can be ~ 1 even for $M_H = 150$ GeV. But for $M_H = 100$ GeV it is always less than 1. For $M_H = 200$ GeV, κ can be as large as ~ 10 , though for very small window in $M_* \sim 1\text{--}1.3$ TeV. Similar variation of κ with M_* is shown in Fig. 4 in the degenerate neutrino mass scenario. In this case, we find that κ can be larger than 1 for $M_H = 200$ GeV. However, as mentioned before, this value of Higgs mass will not serve the purpose of looking for invisible decay modes of the Higgs boson. Hence, for practical purpose, we should look at values of κ for M_H up to 150 GeV. It turns out that for $M_H = 150$ GeV, $\kappa \sim 0.8$ for $M_* \sim 2.5$ TeV, whereas, for $M_H = 100$ GeV, κ can be at most 0.1.

In these two analysis, we have shown that only in a very small region of parameter space can the invisible decay of Higgs be as large as $H \rightarrow b\bar{b}$ decay mode. The main restriction comes from the perturbative constraint on the Yukawa coupling λ .

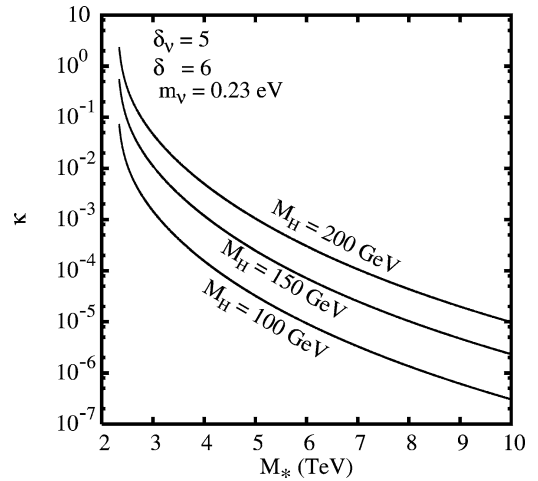


Fig. 4. Variation of κ as a function of M_* in sub-space extra dimensions $\delta_\nu = 5$, number of extra dimensions $\delta = 6$ and $M_H = 100, 150$ and 200 GeV. We have fixed $m_\nu = 0.23 \text{ eV}$.

For completeness we also discuss the case of asymmetric dimensions [5,8]. In this scenario neutrinos propagate in a sub-dimensional space of dimension δ_ν of size R , whereas gravity propagates in space of dimension δ . The extra $(\delta - \delta_\nu)$ has a size r with $(r \ll R)$. In such a scenario Eq. (1) becomes

$$M_{\text{Pl}}^2 \sim R^{\delta_\nu} M_*^{\delta+2} r^{(\delta-\delta_\nu)}. \quad (20)$$

In this case, the Dirac mass for the standard model neutrino becomes

$$m \approx \frac{\lambda v}{\sqrt{2}(M_* R)^{\delta_v}}. \quad (21)$$

To satisfy the constraints from supernova 1987a, the scale $1/R > 10$ KeV [7]. For such a value of $1/R$ the mixing angle ($\sim mR/n$) of any KK state with the standard model neutrinos becomes negligibly small and $m \approx m_\nu$.

One can obtain $m_\nu^2 \sim \Delta m_{\text{atm}}^2$, for $\delta_v = 3$, $\lambda \sim O(1)$, $M_* \approx 4$ TeV (for $R^{-1} \approx 10$ KeV) and $M_* \approx 8$ TeV (for $R^{-1} \approx 25$ TeV). For $\delta_v > 3$, the neutrino mass m_ν is highly suppressed as seen from above expression for m Eq. (21).

In this scenario, the ratio κ as defined in Eq. (13) turned out to be:

$$\kappa \simeq \left(\frac{m^2}{3m_b^2} \right) (M_H R)^{\delta_v}, \quad (22)$$

$$\simeq \left(\frac{\lambda^2 v^2}{6m_b^2} \right) \left(\frac{M_H}{M_*} \right)^{\delta_v}. \quad (23)$$

To study the invisible Higgs decay in this scenario, we fix $\lambda = 1$ and take the same values of Higgs mass as before. We find $\kappa = 0.17, 0.6$ and 1.3 for $M_H = 100, 150$ and 200 GeV, respectively, for $M_* = 1.5$ TeV. From the Eqs. (22) and (23) it is clear that κ decreases as δ_v , and/or M_* increases, while it increases as the higher-dimensional Yukawa coupling λ increases.

Now we will discuss the observability of this kind of invisible decay of some pseudoscalar mesons.

3. Neutral pion decays

We begin our analysis with the neutral pion. The effective Lagrangian for the process $\pi^0 \rightarrow \nu_L \bar{\nu}_R^i$ is in the standard model through Z exchange is:

$$\mathcal{L}_{\text{eff}} = \frac{iG_F}{\sqrt{2}} F_\pi m \bar{\nu} \gamma_5 \nu \Phi_\pi \quad (24)$$

where, G_F and F_π are the Fermi coupling constant and pion decay constant, respectively.

Using this effective Lagrangian we one determine the decay width of pion into $\nu_L \bar{\nu}_R^i$ in the minimal

model:

$$\Gamma(\pi^0 \rightarrow \nu_L \bar{\nu}_R^i) = \frac{G_F^2 F_\pi^2 m^2 m_\pi}{16\pi} \left(\frac{m_\pi}{M_*} \right)^\delta \left(\frac{M_{\text{Pl}}}{M_*} \right)^2. \quad (25)$$

We have computed the branching ratio $\text{BR}(\pi^0 \rightarrow \nu_L \bar{\nu}_R^i)$ for $m_\nu^2 = 4.0 \times 10^{-3} \text{ eV}^2$ and found the result is $\sim 10^{-25}$ for $\delta = 3$, $M_* = 200$ TeV and $\lambda \sim 11$. This predicted branching ratio is much smaller than the experimental upper limit 8.3×10^{-7} at 90% C.L. [17].

The branching ratio can be higher by 4–5 order of magnitude for lower values of $M_* \sim 45$ TeV, but such a low value of M_* correspond to unacceptably large $\lambda \sim 200$. In the case of degenerate neutrinos, the above branching ratio does not change significantly, so we do not present any numerical results here.

Next, we compute the above decay widths in the sub-space scenario. In this scenario as shown earlier one can have λ of order one and also M_* of about few TeV. With the following replacement $\delta \rightarrow \delta_v$ and $(M_{\text{Pl}}/M_*)^2 \rightarrow (M_{\text{Pl}}/M_*)^{2(\delta_v/\delta)}$ one can rewrite the decay width $\Gamma(\pi^0 \rightarrow \nu_L \bar{\nu}_R^i)$ in sub-space model. In this case, we find that for $m_\nu^2 = 4.0 \times 10^{-3} \text{ eV}^2$, the $\text{BR}(\pi^0 \rightarrow \nu_L \bar{\nu}_R^i)$ is of the order 10^{-19} for $M_* \sim 1.4$ TeV corresponding to λ of $O(1)$. The situation remain unchanged even with the assumption of degenerate neutrino masses. In the scenario, where the right-handed neutrinos propagate in the extra dimensions with largest size, the $\text{BR}(\pi^0 \rightarrow \nu_L \bar{\nu}_R^i)$ is too small to be observed.

4. B-meson decays

In the standard model, the process $B \rightarrow \nu \bar{\nu}$ receives contributions from Z -penguin and box diagrams, where the dominant contribution comes from intermediate top quark loop. Off-shell Z and W exchanges have contributions from would be Goldstone modes that couple to right-handed neutrinos. The effective Lagrangian for $B \rightarrow \nu \bar{\nu}$ for each neutrino is given by

$$\mathcal{L}_{\text{eff}} = f_1 \Phi_B \nu \bar{\nu}, \quad (26)$$

where, $f_1 = \frac{G_F}{\sqrt{2}} \frac{\alpha}{2\pi \sin^2 \theta_W} C_{11}^{\text{SM}} V_{td}^* V_{tb} F_B m$, G_F is the Fermi coupling constants, α is the fine structure constant (at the Z mass scale), F_B is the B -meson

decay constant, θ_W is the weak angle and $V_{td}^* V_{tb}$ are products of CKM matrix elements. The Wilson coefficient C_{11}^{SM} at the leading order is given by:

$$C_{11}^{\text{SM}} = \frac{x_t}{8} \left[\frac{x_t + 2}{x_t - 1} + \frac{3(x_t - 2)}{(x_t - 1)^2} \ln(x_t) \right], \quad (27)$$

where $x_t = \frac{m_t^2}{m_W^2}$.

In models of large extra dimension, $B \rightarrow \nu \bar{\nu}$ gets additional contribution from Higgs exchange contribution. The effective flavor changing vertex ($b d H^0$) can be obtained from [18]

$$\begin{aligned} \mathcal{L}_{bdH^0} &= \frac{G_F^{3/2}}{2^{1/4}} \frac{3}{16\pi^2} \sum_i m_i^2 V_{id}^* V_{ib} \\ &\times [m_b \bar{d}(1 + \gamma_5)b + m_d \bar{d}(1 + \gamma_5)b] H^0 + \text{h.c.}, \end{aligned} \quad (28)$$

where, V_{ij} are the elements of the Kobayashi–Maskawa matrix and m_i are the corresponding quark masses flowing in the loop.

The effective Lagrangian for the process $B \rightarrow \nu \bar{\nu}$ retaining only the top quark contribution is:

$$\mathcal{L}_{\text{eff}} = f_2 \Phi_B \bar{\nu}_R^i \nu_L \quad (29)$$

where,

$$f_2 = \frac{G_F}{\sqrt{2}} \left[(\sqrt{2} G_F)^{1/2} \frac{F_B m_B^2}{m_b} \frac{m}{v} V_{td}^* V_{tb} \frac{3m_t^2 m_b}{16\pi^2 M_H^2} \right],$$

M_H is the standard model Higgs mass and v is the standard model vacuum expectation value. The decay width $B \rightarrow \nu_L \bar{\nu}_R^i$ (after summing over all KK modes of right-handed neutrinos below m_B) can be obtained after adding contributions from Z -penguin and box diagrams and the Higgs mediated diagrams together

$$\Gamma(B \rightarrow \nu_L \bar{\nu}_R^i) = \frac{f_{\text{total}}^2}{16\pi} \left(\frac{m_B}{M_*} \right)^\delta \left(\frac{M_{\text{Pl}}}{M_*} \right)^2 m_B, \quad (30)$$

where, $f_{\text{total}} = f_1 + f_2$. To compute the branching ratio we have taken following input parameters $m_b = 4.2$ GeV, $m_B = 5.279$ GeV, $m_t = 175$ GeV, $v = 246$ GeV, $V_{td} = 0.006$, $V_{tb} = 1.0$, $F_B = 0.180$ GeV. We then obtain the $\text{BR}(B \rightarrow \nu_L \bar{\nu}_R^i)$ for case of degenerate neutrinos with $m_\nu = 0.23$ eV (which also corresponds to the heaviest neutrino) and find that the branching ratio varies between $\sim 10^{-12}$ – 10^{-14} for

$M_* \sim 50$ – 120 TeV, $\delta = 3$ and $\lambda \sim 80$ – 20 . This branching ratio is too small to be observed at any present B factories. Even in the sub-space scenario, the branching ratio does not get any significant enhancement, irrespective of the different neutrino masses, assumptions considered previously. In the scenario, where the right-handed neutrinos propagate in the extra dimensions with largest size, the $\text{BR}(B \rightarrow \nu_L \bar{\nu}_R^i)$ is too small to be observed.

5. Conclusions

In this analysis we have studied the possible enhancement of invisible decay widths of the standard model Higgs boson and other pseudoscalar mesons in the model of singlet neutrinos in extra dimensions. In the case of Higgs boson decay we have found that in certain range of extra dimension parameter space, the branching ratio of Higgs into invisible mode can be $\geq \text{BR}(H \rightarrow b\bar{b})$. Unfortunately, the higher-dimensional Yukawa coupling λ takes on large (≥ 100) values in that parameter space. For $\lambda \leq 10$, the $H \rightarrow \nu \bar{\nu}$ rate is a tiny fraction of the $H \rightarrow b\bar{b}$ rate.

We have also studied the invisible decay rate of Higgs in the scenario, where right-handed neutrinos are in sub-space ($\delta_\nu < \delta$), which is the modification of the minimal model, required to keep $\lambda \leq O(1)$. It has been shown that to have a consistent model allowed by different experimental constraint, one should have $\delta_\nu = 5$ and $\delta = 6$. In this scenario, the invisible decay rate of Higgs can compete to that of $H \rightarrow b\bar{b}$, though for a very small range of $M_* \sim 1$ – 1.3 TeV. In the scenario, where the right-handed neutrinos propagate in the extra dimensions with largest size, the invisible decay of Higgs can be as large as $H \rightarrow b\bar{b}$ for $M_H = 200$ GeV, $\delta_\nu = 3$, $\lambda = 1$ and $M_* = 1.5$ TeV. However, for lower values of M_H ($= 100$ and 150 GeV) the invisible decay rate is smaller than $H \rightarrow b\bar{b}$ for the above set of parameters.

We have also studied the decay rates $\pi^0 \rightarrow \nu_L \bar{\nu}_R^i$ and $B \rightarrow \nu_L \bar{\nu}_R^i$ in all these scenarios. Unfortunately in both of these decays the new effects are negligibly small to be measured.

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